

Government Budget Deficits in Large Open Economies

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Abstract

Large and growing levels of public debt in the United States, United Kingdom, Japan and the Euro Area raise new interest in the cross-country effects of a large open economy's deficits. We consider a dynamic optimising model with costly tax collection and exogenously given public spending and initial debt. We ask whether the externalities associated with an individual country's deficits are positive or negative. We characterise the path of taxes in the Nash equilibrium where policy makers act nationalistically and compare this outcome to the global optimal outcome.

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1 Introduction

Net general government debt as a share of GDP is expected to be about 129 percent for Japan, 74 percent for the United Kingdom and 73 percent for the United States in 2011, over twice the levels that prevailed in 2000.¹ This burgeoning official indebtedness raises new interest in a topic that has been relatively neglected in recent years: the cross-country effects of a large country's borrowing. Does such a country's borrowing in an integrated global financial market impose externalities on other countries? If so are, are these spillovers exclusively redistributive or do they create or mitigate inefficiency? In this paper we consider cross-border pecuniary externalities associated with the transmission of national public debt policies through their effect on the global risk-free real interest rate.²

We provide a dynamic equilibrium model with optimising households and governments, in which public debt and the governments' intertemporal budget constraints provide a link between current and future tax decisions. Such models are analytically difficult, especially if they do not exhibit Ricardian equivalence. Thus, our baseline framework possesses the simplest possible supply side for the national economies: a perishable endowment. Our model is inhabited by representative infinite-lived households with log-linear preferences and we assume a simple source of Ricardian non-equivalence, or absence of debt neutrality: increasing and strictly convex real resource costs of administering and collecting taxes. We require that governments follow sustainable plans and we ignore monetary policy; hence, we focus on real interest rate cross-border spillovers that occur in the absence of sovereign default risk and without strategic interactions between a national fiscal authority and a national or supranational monetary authority.

We assume that financial capital is perfectly mobile across countries and that the international transmission of national fiscal policy is solely through interest rates. Taxes

¹International Monetary Fund, World Economic Outlook Database, October 2010.

²Pecuniary externalities are externalities that affect another party solely through the prices faced by the other party.

are lump sum, but because of the strict convexity of the tax collection costs, the timing of taxes matters in this model, just as it would with conventional distortionary taxes on labour income or financial asset income in models with endogenous labour supply or capital accumulation.

Without tax collection costs our model, with its representative households, would exhibit Ricardian equivalence: for any given sequence of public purchases of goods, any sequence of lump-sum taxes and debt that satisfies the intertemporal budget constraint would support the same equilibrium. There would be no international spillovers. This is true even if countries are large in the world capital market and exploit their monopoly power. However, with our strictly convex tax collection costs we demonstrate that if national policy makers cannot cooperate, there are pecuniary externalities that are not merely redistributive and the outcome is inefficient.

If we had instead assumed overlapping generations, then if there were no tax collection costs and taxes were lump sum, alternative rules for financing public purchases of goods would cause purely redistributive pecuniary externalities. Even with symmetric countries, there could be distributional effects between generations, but as long as dynamic inefficiency does not occur, any feasible sequence of lump-sum taxes and debt supports a Pareto efficient allocation.³

In the one-country special case of our model, tax collection costs do not cause inefficiency if one assumes that these costs would be the same in the counterfactual command economy as they are in our market economy. Inefficiency arises when there are multiple countries and each country affects other countries' choice sets through market prices in a way that is not adequately reflected in the market prices.

With symmetric countries and representative, infinite-lived households, symmetric tax policies have no distributional effects. However, because of the convex tax collection costs, they can have welfare consequences if they change the world interest rate. If

³See Buiter and Kletzer (1991). If there is dynamic inefficiency, then fiscal policy that causes redistribution from the young to the old can lead to a Pareto improvement.

countries have outstanding debt and a change in policy causes the interest rate to rise, then higher debt service means that countries must raise taxes, now or in the future, and tax collection costs increase. National governments that maximise their own resident household's welfare do not internalise the cost of the higher tax collection costs to other countries. Thus, if a national government's financing decision raises the world interest rate, then it inflicts a negative externality on the rest of the world. This is in line with conventional wisdom.

Where our model departs from conventional wisdom is through the mechanism by which financing choices affect interest rates. It is conventional – at least in static Keynesian models – to associate deficit financing of public spending with ‘financial crowding out’. That is, for a given public spending programme, larger bond-financed deficits brought about by lower taxes are assumed to raise interest rates.

In our neoclassical intertemporal model this need not be true. Lowering the tax in, say, period zero and raising it in period one so that the government's intertemporal budget constraint is otherwise unchanged in periods two and later results in an increased deficit in period zero, a decreased deficit in period one and a *lower* interest rate on borrowing between periods zero and one. The reason is that lower taxes in period zero and higher taxes in period one result in lower real resource costs associated with tax collection in period zero and higher real resource costs in period one. With exogenously given output this results in higher consumption in period zero and lower consumption in period one. Thus, the price of consumption in period zero relative to the price of consumption in period one falls.

We show that if a government is too small to affect the global interest rate, it minimises the costs of collecting taxes by smoothing them over time. However, if it is able to influence the world interest rate and it has strictly positive initial debt, then it sets a lower tax in the initial period than in future periods. This lowers the interest payment on the government's outstanding debt and, hence, lowers future tax collection costs.

Relative to the global (cooperative) optimum, noncooperative countries tax too much

and issue too little debt in the initial period. Reducing current taxes has a positive welfare spillover, even though it requires issuing more debt. Lowering the current interest rate by lowering current taxes reduces the cost of servicing all countries' outstanding debt and thus reduces all countries' need for costly tax collection. In a noncooperative equilibrium, countries do not take into account this benefit to other countries and they set taxes too high in the initial period.

We demonstrate that as the number of countries goes to infinity and individual countries lose their market power, the noncooperative outcome moves further from the optimal outcome. This result is similar to those obtained by Kehoe (1987) and Chang (1990), but is in marked contrast to those in typical beggar-thy-neighbour policy games.

Our work is related to the vast literature on other types of international fiscal policy linkages. It is also related to the more modest literature on the political economy of the timing of taxes and to the sizable literature on the optimal timing of multiple distortionary taxes.

The literature on the international transmission of fiscal policy has two main strands. First, there is the work on the transmission of government-expenditure shocks with lump-sum taxes and without tax collection costs. Examples are Frenkel and Razin (1985, 1987), Buitier (1987, 1989) and Turnovsky (1988); Turnovsky (1997) provides a survey. The papers in this vein are in contrast to ours in that we take government expenditure as exogenous and ask how the *financing* matters when there are tax collection costs. Second, there are papers on the transmission of tax shocks in models with distortionary taxes in a balanced-budget setting. There is a sizable literature – going back to Hamada (1966) – on the strategic taxation of capital income in a world economy. In this literature, a capital-exporting (importing) country can increase its national income by acting as a monopolist (monopsonist) and restricting capital movements. The result is a Nash equilibrium where nations want to tax capital flows. Other papers consider issues of the feasibility of different tax regimes in an integrated world economy, tax harmonisation and tax competition. Examples of such papers are Sinn (1990) and Bovenberg (1994).

In the political economy literature, excessive public deficits and debt in a closed economy may result from an incumbent government's incentive to signal its competency prior to an election (Rogoff and Sibert (1988)), a war-of-attrition game over the allotment of the costs of fiscal adjustment (Alesina and Drazen (1991)) or a political party's desire to tie the hands of a possible successor (Persson and Svensson (1989) and Alesina and Tabellini (1990)).⁴ Tabellini (1990) extends this latter strand of the literature to a multi-country setting and shows that international cooperation may exacerbate the distortion that generates excessive deficits, thus worsening the outcome. In this paper, we abstract from political economy concerns; at least in the baseline model, governments are able to commit to policies that maximise national welfare.

Chamley (1981, 1986) pioneered the study of dynamic optimal taxation in a closed economy with optimising households when the government can borrow or lend. He focuses on the choice between distortionary capital and labour income taxation and does not consider tax collection costs of the kind studied here. The optimal policy is to impose the maximum possible capital levy on the private sector's initial, predetermined capital and public debt and then to switch permanently to a zero capital income tax rate.⁵ Incorporating tax collection costs of the kind considered here would render Chamley's highly uneven time profile of tax receipts suboptimal.

Our paper analysing the welfare economics of international interest rate spillovers from the tax and borrowing strategies of national governments using a dynamic optimising equilibrium model is perhaps most closely related to Chang (1990,1997). These papers, discussed in more detail in Section 3, are primarily concerned with purely redistributive pecuniary externalities associated with changes in the world real interest rate, however, while our focus is on efficiency losses caused by pecuniary externalities in a model in which, by construction, changes in the world rate of interest have no redistributive impact in equilibrium.

⁴Drazen (2000) provides a discussion of this literature.

⁵See also Lucas (1988).

In Section 2 we present our baseline model where the departure from Ricardian equivalence is driven by tax collection costs, policy makers can commit to future taxes and there are perishable exogenous endowments and log-linear preferences. In Section 3 we discuss the properties of the Nash equilibrium of the game between national policy makers and compare it to the global optimal outcome. In Section 4 we alter the baseline model by assuming that output is endogenous and the result of the households' labour-leisure decisions and that the departure from Ricardian equivalence is a result of distortionary labour taxes. We demonstrate that the global optimal tax policy is qualitatively similar to that of the baseline model. In Section 5 we consider a three-period variant of our baseline model and find the global optimal outcome when it is not possible to commit to future tax policy. We demonstrate that it features taxes that rise over time. In Section 6 we consider a production economy with capital accumulation, CES preferences and costly tax collection. We show that if the world is at a steady state with strictly positive debt, then a coordinated reduction in current taxes financed by higher future taxes improves welfare. Section 7 is the conclusion.

2 The Model

The model comprises $N \geq 1$ countries, each inhabited by a representative infinite-lived household and a government. Each period, each household receives an endowment of the single tradable, non-storable consumption good and each government purchases an exogenous amount of the good. Governments finance their purchases by issuing debt or by taxing their resident households. We assume that the tax system is costly to administer; governments use up real resources collecting taxes. All saving is in the form of privately or publicly issued real bonds. We assume that the endowments and the governments' purchases are constant over time and identical across countries. The households' preferences and initial asset holdings and the governments' initial debt are also identical across countries. There is perfect capital mobility, and hence, a common world interest rate.

2.1 The households

The country- i household, $i \in \mathbb{Z}_N$, has preferences over its consumption path given by

$$u^i = \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \quad (1)$$

where c_t^i is its period- t consumption and $\beta \in (0, 1)$ is its discount factor.⁶ The household's period- t budget constraint is

$$c_t^i + a_{t+1}^i = W - \tau_t^i + R_t a_t^i, \quad t \in \mathbb{Z}_+, \quad (2)$$

where a_t^i is the household's holdings of real bonds at the start of period t , $W > 0$ is its per-period endowment of the good, τ_t^i is its period- t tax bill and R_t is (one plus) the interest rate between periods $t - 1$ and t . The household's initial assets, a_0 , are given.

In addition to satisfying its within-period budget constraint, the household must satisfy the long-run solvency condition that the present discounted value of its assets is not strictly negative as time goes to infinity. The transversality condition associated with its optimisation problem ensures that the present discounted value of its assets is not strictly positive. Thus,

$$\lim_{t \rightarrow \infty} a_{t+1}^i / \rho_t = 0, \quad (3)$$

where $\rho_t \equiv \prod_{s=0}^t R_s$ is (one plus) the interest rate between periods 0 and t .

Equations (2) and (3) imply that the present discounted value of the household's consumption equals the present discounted value of its disposable endowment income plus its initial assets:

$$\sum_{t=0}^{\infty} (w_t - \tau_t^i) / \rho_t = \sum_{t=0}^{\infty} c_t^i / \rho_t, \quad \text{where } w_t \equiv \begin{cases} W + R_0 a_0 & \text{if } t = 0 \\ W & \text{otherwise.} \end{cases} \quad (4)$$

The household chooses its consumption path to maximise its utility function (1) sub-

⁶We use the notation $\mathbb{Z}_N \equiv \{1, 2, \dots, N\}$, $\mathbb{Z}_+ \equiv \{0, 1, \dots\}$ and $\mathbb{Z}_{++} \equiv \{1, 2, \dots\}$.

ject to its intertemporal budget constraint (4). The solution satisfies the Euler equation

$$c_{t+1}^i = \beta R_{t+1} c_t^i, \quad t \in \mathbb{Z}_+. \quad (5)$$

Solving the difference equation (5) yields the household's period- t consumption as a function of its initial consumption and the interest rate between periods 0 and t :

$$c_t^i = \beta^t \rho_t c_0^i / R_0, \quad t \in \mathbb{Z}_+. \quad (6)$$

Substituting equation (6) into equation (4) yields the household's initial consumption as a function of its taxes and the interest rates:

$$c_0^i = (1 - \beta) \rho_0 \sum_{t=0}^{\infty} (w_t - \tau_t^i) / \rho_t. \quad (7)$$

Substituting equation (6) into equation (1) yields the household's indirect utility as a function of initial consumption and the interest rates:

$$u^i = \ln c_0^i + (1 - \beta) \sum_{t=1}^{\infty} \beta^t \ln \rho_t. \quad (8)$$

Here and throughout the paper we ignore constants that do not affect the household's optimisation problem.

2.2 The government

The country- i , $i \in \mathbb{Z}_N$, government's period- t budget constraint is

$$\tau_t^i - \phi (\tau_t^i)^2 / 2 + b_{t+1}^i = G + R_t b_t^i, \quad t \in \mathbb{Z}_+, \quad (9)$$

where b_t^i is the government's outstanding debt at the start of period t and $G > 0$ is its per-period purchase of the good. The tax collection cost associated with a tax τ is $\phi \tau^2 / 2$,

$\phi > 0$.⁷ We allow for negative taxes, or subsidies. In this case the collection cost is the cost of administering and disbursing the surplus. We discuss the tax collection cost in more detail in the next subsection.

The government's initial debt, b_0 , is given and we restrict ourselves to the empirically relevant case of $b_0 \geq 0$. We restrict the model's parameters so that satisfying equation (9) is feasible; the restrictions are detailed later in this section.

In addition to satisfying its within-period budget constraint, the government also satisfies

$$\lim_{t \rightarrow \infty} b_{t+1}^i / \rho_t = 0. \quad (10)$$

As with the household, this is an implication of the long-run solvency constraint and the transversality condition associated with the government's optimisation problem.⁸

Equations (9) and (10) imply that the present discounted value of the government's purchases, plus its initial debt, equals the present discounted value of its tax stream, net of collection costs:

$$\sum_{t=0}^{\infty} \left[\tau_t^i - \phi (\tau_t^i)^2 / 2 - g_t \right] / \rho_t = 0, \text{ where } g_t \equiv \begin{cases} G + R_0 b_0 & \text{if } t = 0 \\ G & \text{otherwise.} \end{cases} \quad (11)$$

2.3 Market clearing

Market clearing requires that the sum of the N households' asset holdings equals the sum of the N governments' debt. Thus,

$$a_t = b_t, \quad t \in \mathbb{Z}_+, \quad (12)$$

⁷Barro (1979) pioneered these strictly convex tax collection costs in a closed-economy setting. He assumed that the government minimises the discounted sum of these costs, rather than maximises the discounted welfare of households.

⁸We ignore the important issue of the possibility of sovereign default. There is already a sizable literature on this subject. See, for example, Eaton and Gersovitz (1981), Bulow and Rogoff (1989) and Cole and Kehoe (1995).

where variables without a superscript denote global averages. By equation (12), w_0 , defined in equation (4), is equal to $W + R_0 b_0$.

Goods market clearing requires that the sum of average household consumption, average government purchases and average tax collection costs equals the average endowment. Thus,

$$c_t = W - G - \frac{1}{N} \sum_{j=1}^N \frac{\phi(\tau_t^j)^2}{2}, \quad t \in \mathbb{Z}_+. \quad (13)$$

Equation (13) is, of course, also implied by equations (2), (9) and (12).

Averaging both sides of the Euler equation (6) over the N countries yields

$$\rho_t = \rho_0 c_t / (\beta^t c_0), \quad t \in \mathbb{Z}_+. \quad (14)$$

Equations (13) and (14) imply that in equilibrium, the interest rate between periods 0 and t is solely a function of period-0 and period- t taxes.

Lower period-0 taxes financed by higher period- t taxes *lower* the interest rate between periods 0 and t . Before continuing, it is interesting to ask whether the timing of taxes affects the global risk-free interest rate in the manner that this model predicts. There is a large body of empirical work attempting to quantify the relationship between government budget deficits and interest rates. However, the results are mixed, the literature is problematic and the results are hard to interpret for several reasons.⁹

First, both taxes and interest rates are endogenous and an apparent relationship between them may be due to the influence of other variables. For example, automatic stabilisers cause tax revenue to be lower and deficits to be higher during recessions. At the same time, an expansionary monetary policy (not considered in this paper) may temporarily lower real interest rates. Thus, the role of monetary policy over the business cycle may cause deficits and real interest rates to be negatively correlated. Second, while lowering taxes may lower the global risk-free interest rate, it may also increase sovereign-default risk premia. This is ruled out in our model because each government is assumed

⁹See Baldacci and Kumar (2010) for a discussion of the results and problems.

to satisfy its solvency constraint. If the effect on the national sovereign risk premium is larger than the effect on the global risk-free interest rate, it will cause tax decreases to be associated with higher measured market interest rates. Third, if tax cuts in a country include lower capital taxes, then the country's marginal product of capital may fall to equate after-tax returns across countries. This causes lower taxes to be associated with lower (before-tax) interest rates.

Finally, the model here predicts that lowering taxes in the current period and raising them next period increases the current deficit and lowers the interest rate. However, it also predicts that lowering current and future taxes by the same amount increases the current deficit and has no effect on the interest rate; lowering the current tax by less than next period's tax is lowered increases both the current deficit and the interest rate. In empirical studies, it is difficult to control for the public's expectation of future tax policy.

Substituting equation (14) into equations (7) and (8) yields

$$c_0^i = (1 - \beta) c_0 \sum_{t=0}^{\infty} \beta^t (w_t^i - \tau_t^i) / c_t. \quad (15)$$

$$u^i = \ln c_0^i - \beta \ln c_0 + (1 - \beta) \sum_{t=1}^{\infty} \beta^t \ln c_t. \quad (16)$$

Substituting equation (15) into equation (16) yields

$$u^i = \ln \left(\sum_{t=0}^{\infty} \beta^t (w_t - \tau_t^i) / c_t \right) + (1 - \beta) \sum_{t=0}^{\infty} \beta^t \ln c_t. \quad (17)$$

Substituting equation (14) into equation (11) yields

$$B^i \equiv \sum_{t=0}^{\infty} \beta^t s_t^i = 0, \text{ where } s_t^i \equiv \left[\tau_t^i - \phi (\tau_t^i)^2 / 2 - g_t \right] / c_t. \quad (18)$$

Substituting equation (13) into equations (17) and (18) would allow both the household's indirect utility and the government's budget constraint to be expressed solely as functions of the paths of the taxes in the N countries.

For $t > 0$, s_t^i in equation (18) is the government's period- t primary budget surplus (or minus one times the deficit, if negative), divided by c_t . For $t = 0$, s_0^i is the primary surplus less principal and interest on the outstanding debt divided by c_0 . As a convenient, if somewhat inelegant shorthand, we will refer to s_t^i as country i 's period- t *discounted surplus* and we will refer to $\tau_t^i - \phi(\tau_t^i)^2/2 - g_t$ as country i 's period- t *surplus*, $t \in \mathbb{Z}_+$.

2.4 Taxes and Revenues

In this subsection we describe some of the properties of the surplus function. We define the set of feasible taxes and impose some restrictions on the parameter space. We derive some results about the discounted surplus functions. Finally, we discuss some of the empirical results relating to real world tax collection costs.

The surplus curve $\tau_t - (\phi/2)\tau_t^2 - g_t$ looks like a Laffer curve, although its shape is the result of tax collection costs rather than the distortions associated with non-lump-sum taxes. It attains a maximum of $1/(2\phi)$ at $\tau = 1/\phi$. Temporarily supposing that they exist, denote the two taxes that yield a surplus of zero by τ_t^- and τ_t^+ , where $0 < \tau_t^- < 1/\phi < \tau_t^+$.¹⁰ There is a strictly positive surplus in period- t in country i if and only if $\tau_t^i \in (\tau_t^-, \tau_t^+)$.

A set of period- t taxes $\{\tau_t^i\}_{i \in \mathbb{Z}_N}$ is said to be *feasible* if they leave consumption in each country strictly positive. Let $\bar{\tau}$ be the least upper bound on the set of feasible taxes in a symmetric outcome.¹¹ We assume that the surplus maximising tax is feasible, but that a tax equal to the entire endowment is not. To ensure the existence of an equilibrium where the solution to the policy maker's problem is interior it is simple and sufficient – but not necessary – to impose conditions that imply that the government can always run a surplus and that the government can satisfy its intertemporal budget constraint even

¹⁰Clearly $\tau_t^- = (1/\phi)(1 - \sqrt{1 - 2\phi g_t})$ and $\tau_t^+ \equiv (1/\phi)(1 + \sqrt{1 - 2\phi g_t})$, $t \in \mathbb{Z}_+$.

¹¹By equation (13), $\bar{\tau} \equiv \sqrt{(2/\phi)(W - G)}$.

if its period-0 tax is zero. Thus,¹²

$$1/\phi < \bar{\tau} < W, \quad 1/(2\phi) > g_0, \quad \beta \geq 1/(1 + \alpha), \quad (19)$$

where $\alpha \equiv \frac{1/(2\phi) - G}{g_0} \frac{\phi \bar{\tau}^2}{\phi \bar{\tau}^2 - 1}$.

In the following proposition we show that, given the taxes of other countries, each country's discounted surplus as a function of its own tax has a similar appearance to its surplus function. However, the tax that maximises the discounted surplus is greater than the tax that maximises the surplus. Moreover, in a multi-country world, if taxes are identical across countries and if each country's tax maximises its discounted surplus, then each country's tax is less than the tax that maximises the discounted surplus when there is only one country.

Proposition 1 *Let $t \in \mathbb{Z}_+$, $i \in \mathbb{Z}_N$ and suppose that $\tau_t^j < \bar{\tau}$, $j \neq i$. Then, s_t^i has a unique maximum $\tau_{Nt}^{i*} \in (1/\phi, \tau_t^+)$ and is strictly increasing in τ_t^i on $[0, \tau_{Nt}^{i*})$ and strictly decreasing on $(\tau_{Nt}^{i*}, \tau_t^+]$. Furthermore, if taxes are identical across countries then $\tau_{Nt}^* < \tau_{1t}^*$ if $N > 1$.*

Proof. All proofs are in the Appendix. ■

The intuition for why a country can increase its discounted surplus by increasing taxes above the tax that maximises its surplus is as follows. Suppose that a country is maximising its surplus. Then a marginal increase in its tax has no effect on the surplus, which is insensitive to taxes at that level, but it lowers average consumption and thus increases the discounted surplus. If there is only one country, then the impact of this tax increase on average consumption is greater than the impact of an increase in a single country's tax in a multi-country world. Thus, the discounted surplus maximising tax is higher when there is just one country than when there are multiple countries and countries act symmetrically.

¹²Proposition 1 and $1/(2\phi) > g_0$ will ensure that $\alpha > 0$ and, hence, it is always possible to find such a $\beta \in (0, 1)$.

We have modelled the cost of collecting taxes as an administrative tax; we could have modelled it as a compliance cost. These interpretations are equivalent as in either case the tax is ultimately borne by the households. Before proceeding to the policy makers' optimisation problem it is reasonable to discuss the empirical evidence on the importance of such costs.

The OECD (2009) estimates that administrative costs as a percentage of resulting net tax revenue were 1.53 for Japan, 1.10 for the United Kingdom and .45 for the United States in 2007. Estimating compliance costs is more difficult and most studies have focused on a particular component. Sanford et al (1989) estimates the compliance cost as a share of revenue for UK corporate taxes to be 2.22 percent. Allers (1995) estimates the compliance cost as a share of revenue for Netherlands corporate taxes to be about four percent. Slemrod and Venkatesh (2002) estimate the compliance cost as a share of revenue for medium-size US firms to be 28.0 - 29.6 percent. Pitt and Slemrod (1989) estimate that in 1982 it cost the average itemising US tax payer \$43 dollars to itemise deductions; Guyton et al (2003) estimate that in 2003 the overall compliance cost of the average US tax payer was 25.5 hours and \$149. Slemrod (1996) estimates overall compliance costs to be about ten percent of resulting revenue. Slemrod and Sorum (1984) estimate the total resource cost associated with administration and compliance in the United States to be seven percent of the resulting revenue: about twice as high as the efficiency cost associated with tax distortions.¹³

While there appears to be substantial evidence that tax collection costs are both absolutely large and large relative to the costs arising from tax distortions, there is little empirical evidence about the shape of the tax collection function. Nevertheless it seems reasonable that the cost increases at an increasing rate as attempts to collect increasing amounts of tax lead to increasing efforts to evade or avoid such revenue collection. The assumption that costs are convex is common in the public finance literature. See, for

¹³One might add to this tax litigation costs, psychological costs resulting from anger, dissatisfaction and frustration at the tax system, as well as real resource costs arising from disruptive protests and economic unrest.

example, Wilson (1989), Kaplow (1990) and Aragón (2010).

3 Dynamic Optimal Taxation

We assume that in period 0 the government in country i can commit to a tax plan $\{\tau_t^i\}_{t=0}^\infty$. It takes the tax plans of the other governments as given and chooses feasible taxes to maximise the indirect utility of its household (equation (17)) subject to its budget constraint (equation (18)), where average global consumption is given by (13). We consider Nash outcomes where countries act symmetrically.

We first show that subsidies cannot be part of a symmetric Nash equilibrium and that taxes must be no greater than the ones that maximise the discounted surpluses.

Proposition 2 *A symmetric Nash equilibrium must have $\tau_t \in [0, \tau_{Nt}^*]$, $t \in \mathbb{Z}_+$.*

The intuition is straightforward. If a country were to choose a tax that is greater than the one that maximises the discounted surplus, then by Proposition 1, this tax is on the "wrong" side of the country's discounted surplus Laffer curve. Thus, there is a tax on the "right" side of this Laffer curve that yields the same discounted surplus and produces a smaller distortion. If a country were to provide a subsidy in some period, then it must impose a tax in some other period. If the country were to lower the subsidy it would both increase consumption in that period and improve the fiscal situation, allowing taxes to be lowered in other periods.

We next show that a unique equilibrium exists and we describe the time path of equilibrium taxes.

Proposition 3 *There exists a unique symmetric Nash equilibrium and it has constant taxes from period 1 on. Furthermore, if initial government debt is strictly positive, equilibrium taxes are lower in period 0 than in period 1. If there is no initial government debt, taxes are constant over time.*

The intuition behind the result is that the government trades off two objectives. First, it wants to smooth consumption by smoothing tax collection costs over time. If this were

its sole objective, optimality would require constant taxes. Second, however, it wants to lower the discounted value of the tax collection costs through its influence on the global interest rate. It does this by lowering initial taxes and raising future taxes. Through the goods market clearing condition (equation (13)) this raises initial consumption and lowers future consumption. By the Euler equation (14) this lowers the interest rate between periods 0 and 1. Thus its required tax revenue falls and so do its tax collection costs.

Proposition 4 *If countries have no market power ($N \rightarrow \infty$) then taxes are constant across periods.*

When countries have no market power, we have Barro's (1979) result. If these costs are convex, then an optimising government smooths them over time.

We now compare the equilibrium and optimal outcomes and show how the number of countries affects the deviation of the equilibrium outcome from the global optimal outcome. We note that the outcome when $N = 1$ is the global optimal outcome.

Proposition 5 *Suppose that $N > 1$. If there is strictly positive initial government debt, then the period 0 tax is too high relative to the global optimal tax and subsequent taxes are too low. Furthermore, increasing the number of countries increases the difference between equilibrium taxes and global optimal taxes.*

The intuition for the first part of the proposition is that with strictly positive initial debt, lowering the period-0 tax below subsequent taxes causes a positive externality by decreasing all countries' borrowing costs. Countries do not take into account this social benefit and they do not lower period-0 taxes enough. The intuition for the second part of the proposition is that as the number of countries increases and the effect of any one country on global variables declines, the failure of countries to take into account the effect of their actions on the world economy becomes more severe.

Our paper can be contrasted to those of Chang (1990,1997), who also models government debt as a dynamic game between national governments. In his overlapping-

generations framework two-period-lived households produce output when young and consume when old. As no household consumes in more than one period, the Euler equation of our paper – which results in the real interest rate playing the role of the price of consumption today relative to consumption tomorrow – is replaced by a static efficiency condition that output, and equivalently saving, is increasing in the real interest rate. Thus an increase in the deficit requires an increase in the real interest rate to increase saving and restore equilibrium.

Unlike in our model, a higher real interest rate (brought about by lower taxes, rather than by higher taxes as in our model), does not inflict a real resource cost on the rest of the world. Because taxes are lump sum and there are no tax collection costs, the pecuniary externality in Chang’s model associated with the effect of a country’s tax policy on the global real interest rate is purely redistributive. Even though all feasible national tax policies in Chang’s model support equilibria that are both dynamically and Pareto efficient, they are welfare ranked by the nationalistic governments who maximise a discounted sum of the welfares of current and future generations of residents. The non-cooperative equilibria have larger government deficits and higher interest rates than the cooperative equilibria.

Our result that the equilibrium outcome is furthest from the global optimal outcome when the number of countries goes to infinity and countries lose their market power is similar to Chang’s (1990) result that the non-cooperative solution is furthest from the cooperative solution when the number of countries go to infinity and is in stark contrast to the result in “beggar-thy-neighbour” policy games where nations attempt to exploit their market power to gain at the expense of other countries. In such papers, as the number of countries goes to infinity and nations lose their market power, the noncooperative outcome converges to the cooperative outcome.¹⁴

¹⁴This would occur for, for example, in Hamada (1966).

4 Conventional Tax Distortions

In the previous section we considered a scenario where the departure from Ricardian equivalence is due to tax collection costs. We chose this framework as our baseline both for its relative tractability and because we believe that tax collection costs are empirically more important than the efficiency and welfare costs associated with conventional tax distortions. However, as it is more conventional in the economics literature to focus on the latter friction, in this section we consider a one-country model where the departure from Ricardian equivalence is caused by distortionary taxes rather than by tax collection costs.

4.1 The model with tax distortions

We assume that the household is endowed with one unit of time which it allocates between labour and leisure. It can produce output one-for-one from labour and it pays a proportional labour-income tax τ_t on its output. Its saving is in the form of real interest-bearing bonds.

In each period $t \in \mathbb{Z}_+$ the household receives utility from its consumption of the good and of leisure, $1 - l_t$:

$$u = \sum_{t=0}^{\infty} \beta^t [\alpha \ln c_t + (1 - \alpha) \ln (1 - l_t)], \quad \alpha \in (0, 1). \quad (20)$$

It maximises equation (20) subject to the within-period budget constraints

$$c_t + a_{t+1} = (1 - \tau_t) l_t + R_t a_t, \quad t \in \mathbb{Z}_+ \quad (21)$$

and the terminal condition (3), taking a_0 as given.

Necessary and sufficient conditions for optimality are equations (3) and (21), the static optimality condition

$$1 - l_t = \frac{(1 - \alpha) c_t}{\alpha (1 - \tau_t)} \quad (22)$$

and the (one-country) Euler equation (14).

The government satisfies the within-period budget constraint

$$\tau_t l_t + b_{t+1} = G + R_t b_t, \quad t \in \mathbb{Z}_+ \quad (23)$$

and the terminal condition (10), taking b_0 as given. Solving equation (23) forward and substituting (10) into the result yields the government's intertemporal budget constraint

$$\sum_{t=0}^{\infty} (\tau_t l_t - g_t) / \rho_t = 0. \quad (24)$$

Substituting equation (22) into equation (24) and equation (14) into the result yields

$$\sum_{t=0}^{\infty} \frac{\tau_t - g_t}{\rho_t} - \frac{(1 - \alpha) c_0}{\alpha R_0} \sum_{t=0}^{\infty} \frac{\beta^t \tau_t}{1 - \tau_t} = 0. \quad (25)$$

Market clearing requires that the bond markets clear (equation (12)) and that goods markets clear

$$c_t + G = l_t, \quad t \in \mathbb{Z}_+. \quad (26)$$

Substituting equation (22) into equation (26) and equation (14) into the result yields

$$c_0 \rho_t = \frac{\alpha R_0 (1 - G)}{\beta^t h_t}, \quad t \in \mathbb{Z}_+, \quad \text{where } h_t \equiv \frac{1 - \alpha \tau_t}{1 - \tau_t}. \quad (27)$$

This implies

$$\rho_t = \frac{R_0 h_0}{\beta^t h_t}, \quad t \in \mathbb{Z}_+. \quad (28)$$

As in the previous section, a decrease in the period-0 tax financed by an increase in the period- t tax lowers the interest rate between periods 0 and t .

4.2 Optimal Taxes

Substituting equation (22) into equation (20) and using equations (14) and (27) yields

$$u = \sum_{t=0}^{\infty} \beta^t [\alpha \ln(1 - \tau_t) - \ln(1 - \alpha\tau_t)]. \quad (29)$$

Substituting equations (27) into equation (25) yields

$$B \equiv \alpha(1 - \beta) \sum_{t=0}^{\infty} \beta^t \tau_t - G - (1 - \beta) R_0 h_0 b_0 = 0. \quad (30)$$

As in Section 3, the policy maker chooses the path of taxes to maximise indirect utility (29) subject to the budget constraint (30). As in Section 2, we need to make assumptions that ensure an interior optimum exists. We assume that

$$G < \alpha, \quad R_0 b_0 \leq \min \{ \alpha(1 - \beta) / (1 - \alpha), \alpha\beta - G, (\alpha - G) / \alpha \}. \quad (31)$$

These inequalities are convenient and sufficient to ensure that it is possible to satisfy the government's intertemporal budget constraint with both a constant tax and $\tau_0 = 0$.

We demonstrate that there is a unique solution to the policy maker's problem and we describe the time path of the optimal taxes.

Proposition 6 *There exists a unique optimum and it has the property that $\tau_t = \tau_1 > \tau_0$, $t \in \mathbb{Z}_{++}$.*

As in the previous section, global optimality has lower taxes initially and higher taxes later on.

It appears to be impossible (at least to us) to analyse the Nash equilibrium for the case of $N > 1$ analytically. However, it is easy to show that if $N \rightarrow \infty$, then the Nash equilibrium has constant taxes. With no market power, the sole goal of policy makers is to smooth distortions over time. Hence, countries without market power set the period-0 tax too high and the subsequent taxes too low, relative to the global optimum.

5 Time Consistent Taxes

The results in Sections 3 and 4 depend on the assumption that the government can commit to a path of planned taxes. Proposition 3 says that the period-0 tax is lower than subsequent taxes. This implies that the government enters period 1 with strictly positive debt. Thus, if the government could re-optimize in period 1, Proposition 3 would imply that it would set a lower tax in period 1 than in later periods. This implies that the equilibrium, which features constant taxes from period 1 on, is not time consistent unless there is no initial debt or countries have no market power. In this section we consider a three-period variant of the baseline model and find the time-consistent taxes for the case of a single country.

The household's lifetime utility is given by $\ln c_0 + \beta \ln c_1 + \beta^2 \ln c_2$. The household's budget constraint is given by equation (2) for $t = 0, 1, 2$, where $a_3 = 0$ and a_0 is given. Optimality for the household requires that the Euler equation (5) hold for $t = 0, 1$. The government budget constraints are given by (9) for $t = 0, 1, 2$, where $b_3 = 0$ and b_0 is given. Market clearing requires that equation (12) holds for $t = 0, 1, 2$. Consumption is given by the goods market clearing condition (13) for $t = 0, 1, 2$.

To find the time-consistent solution we work backwards, first maximising utility starting in period 1 and taking $R_1 b_1$ as given. This gives τ_1 and τ_2 , as well as maximised utility from period 1 on as functions of $R_1 b_1$. In period 1, the government takes $R_0 b_0$ and the functional forms of τ_1 , τ_2 and maximised utility from period 1 on as given. Using the Euler equation between period 0 and period 1, we can find $R_1 b_1$ as a function of τ_0 and b_1 . Thus, maximised utility from period 1 on can be expressed as a function solely of the τ_0 and b_1 . The policy maker then chooses τ_0 and b_1 to maximise lifetime utility subject to the period-0 budget constraint. Leaving the technical details for the appendix, we have the following result.

Proposition 7 *If there is strictly positive initial debt, then time-consistent taxes are strictly increasing over time.*

This section demonstrates that the result from the baseline model that global optimal taxes are constant from period 1 on depends on the policymaker's ability to commit to planned future taxes. However, the base-line model's result that global optimal taxes should be relatively low in period-1 and higher thereafter does not depend upon an ability to commit.

6 Production and Capital Accumulation

An important simplifying feature of the baseline model is that varying the timing of taxes, and thus tax collection costs, over time is the only way to transfer real resources across periods. In equilibrium, net global private and public saving is always zero because the good is perishable. Reducing taxes in any given period increases the resources available that period and increases private consumption. In this section, we allow for production using capital as an input. Thus, real resources can be transferred across periods not only by changing the path of taxes, but also by capital formation.

In this section we assume that there are N countries. We assume that the household in country $i \in \mathbb{Z}_N$ has CES preferences

$$u^i = \frac{1}{1-\theta} \sum_{t=0}^{\infty} \beta^t \left[(c_t^i)^{1-\theta} - 1 \right], 0 < \beta < 1, 0 < \theta \neq 1, \quad (32)$$

where θ is the reciprocal of the elasticity of intertemporal substitution. As $\theta \rightarrow 1$, the above preferences become the logarithmic specification of Section 2.

The single good in the model is both a capital and a consumption good. The representative households each supply one unit of labour inelastically each period and save both bonds and the output of the current good in the form of capital. The saved capital is loaned to the firms to be used in the next-period's production process. The firms transform capital and labour into output via a Cobb-Douglas production function where output per unit of labour is $f(k) = Ak^\alpha$, where k is the capital-labour ratio, $A > 0$ and $\alpha \in (0, 1)$. We suppose that labour is immobile across countries, capital is perfectly

mobile and capital depreciates completely. Then perfect mobility of capital and perfect competition imply that capital-labour ratios and wages are equalised across countries and $k_t = k(R_t) = [A(1 - \alpha)/R_t]^{1/\alpha}$.

The Euler equation of the household's optimisation problem becomes

$$c_{t+1}^i = (\beta R_{t+1})^{1/\theta} c_t^i, \quad t \in \mathbb{Z}_+. \quad (33)$$

Solving the difference equation (33) and averaging across countries yields

$$\rho_t = (1/\beta^t) (c_t/c_0)^\theta, \quad t \in \mathbb{Z}_{++}. \quad (34)$$

The government's budget constraint is given by equation (11). Substituting equation (34) into equation (11) yields

$$\sum_{t=0}^{\infty} \beta^t c_t^{1-\theta} s_t^i = 0. \quad (35)$$

Market clearing requires (12) and

$$f(k(R_t)) - G - (\phi/2) \sum_{i=1}^N (\tau_t^i)^2 - c_t - k_{t+1} = 0. \quad (36)$$

The model with capital is far more difficult to analyse than the one without. To obtain an analytical result, we restrict ourselves to a simple experiment.

Proposition 8 *Suppose countries are at a symmetric steady state with constant taxes and strictly positive public debt. Then it is possible to increase welfare with a coordinated marginal tax cut in the current period.*

The proof demonstrates that welfare can be improved with a current tax cut (and associated current consumption increase) financed by a constant permanent future tax rise that reduces future consumption by a constant amount.

Lowering the current tax and raising future taxes raises current consumption and

lowers future consumption, thus lowering the current interest rate as in the previous sections. This lowers the cost of servicing the debt and reduces future tax collection costs. To see that the interest rate must fall, suppose that it did not. Then next period's marginal product of capital rises and current capital accumulation falls. With lower tax collection costs and fixed current output, this implies current consumption rises. This is inconsistent with the interest rate falling in the current period unless next period's, and hence every future period's, consumption rises by more than current consumption. However, with lower current capital accumulation and higher future tax collection costs this is impossible.

7 Conclusion

We have demonstrated that, in our baseline model with costly tax collection, optimising governments will perfectly smooth taxes if they have no market power or no initial debt. If countries are large enough to affect the world interest rate and they have strictly positive initial debt, then optimising national governments will set lower taxes in the current period than in the future. We show that, relative to the global optimal outcome, non-cooperative national governments set current taxes too high and future taxes too low. Thus, relative to the optimum, initial budget deficits are too low and future deficits are too high.

We extend our baseline model to consider a departure from Ricardian equivalence caused by distortionary labour taxes and show that the optimal outcome is similar to that in the baseline model: the period-0 tax is lower than the taxes from period 1 on. We consider a three-period model where commitment to future taxes is not possible and demonstrate that the cooperative time-consistent taxes are rising over time. Finally, we consider a model with CES preferences and capital accumulation. We demonstrate that if the world is in a steady state with strictly positive government debt, then welfare can be increased with a current tax cut financed by higher future taxes.

8 Appendix

Proof of Proposition 1. By (13) and the definition of s_t^i in (18)

$$\partial s_t^i / \partial \tau_t^i = (1 - \phi \tau_t^i + \phi \tau_t^i s_t^i / N) / c_t. \quad (37)$$

We have that $\partial s_t^i / \partial \tau_t^i$ is continuous in τ_t^i on $[0, \tau_t^+]$ with $\partial s_t^i / \partial \tau_t^i = (1 - \phi \tau_t^+) / c_t < 0$ at $\tau_t^i = \tau_t^+$ and $\partial s_t^i / \partial \tau_t^i = s_t^i / (N c_t) > 0$ at $\tau_t^i = 1/\phi$. Thus, s_t^i has a critical point τ_{Nt}^{i*} in $(1/\phi, \tau_t^+)$. If $\tau_t^i \in [0, \tau_t^+]$ then $\partial s_t^i / \partial \tau_t^i = 0 \Rightarrow \partial^2 s_t^i / \partial \tau_t^{i2} = -\phi(1 - s_t^i / N) / c_t = -(c_t \tau_t^i)^{-1} < 0$. Thus, τ_{Nt}^{i*} is the unique maximiser of s_t^i in $[0, \tau_t^+]$ and $\partial s_t^i / \partial \tau_t^i > 0$ if $\tau_t^i \in [0, \tau_{Nt}^{i*})$ and $\partial s_t^i / \partial \tau_t^i < 0$ if $\tau_t^i \in (\tau_{Nt}^{i*}, \tau_t^+]$. As $s_t^i < 0$ if $\tau_t^i \notin [0, \tau_t^+]$, τ_{Nt}^{i*} is the unique maximiser on $(-\bar{\tau}, \bar{\tau})$.

By (13), (37), the definition of s_t^i and the definitions of $\bar{\tau}$, w_t and g_t (from (4) (with $a_0 = b_0$), (11) and footnote 11), $\partial s_t^i / \partial \tau_t^i = 0$ when $\tau_t^i = \tau_t$, $i \in \mathbb{Z}_N$, implies $\bar{\tau}^2 - 2w_t \tau_t + \tau_t^2 = 2\tau_t s_t c_t (N - 1) / N$. The left-hand side of this equation is strictly decreasing on $(1/\phi, \tau_t^+)$. When $N = 1$ the right-hand side is zero and when $N > 1$ the right-hand side is strictly positive for $\tau_t \in (1/\phi, \tau_t^+)$. Hence, $\tau_{Nt}^* < \tau_{1t}^*$.

Note that $\tau_{Nt}^* = \tau_{N1}^*$, $t \in \mathbb{Z}_{++}$ and $\tau_{1t}^* = w_t - \sqrt{w_t^2 - \bar{\tau}^2}$.

Proof of Proposition 2. We first show that no symmetric optimum can have taxes greater than τ_{Nt}^* . Suppose to the contrary that there exists a symmetric equilibrium where $\exists t$ such that $\tau_t = \tau^W > \tau_{Nt}^*$. The continuity of s_t^i in τ_t^i on the set of feasible taxes ensures that there exists a feasible $\tau^R < \tau_{Nt}^*$ such that s_t^i at τ^R equals s_t^i at τ^W . Thus, by (18), a switch from $\tau_t^i = \tau^W$ to $\tau_t^i = \tau^R$ does not require a change in any other tax. The government prefers $\tau_t^i = \tau^R$ to $\tau_t^i = \tau^W$ if its indirect utility (17) is greater at $\tau_t^i = \tau^R$ than at $\tau_t^i = \tau^W$. Let c_t evaluated at $\tau_t^i = \tau^K$ be denoted by c_t^K , $K = W, R$. Then this is the case if $c_t^R > c_t^W$ and $(w_t - \tau^R) / c_t^R > (w_t - \tau^W) / c_t^W$. The first inequality is clearly true. By the definitions of w_t and g_t and s_t^i , the second inequality is true iff $\left[W - G - (\phi/2) (\tau^R)^2 - c_t^R s_t \right] / c_t^R > \left[W - G - (\phi/2) (\tau^W)^2 - c_t^W s_t \right] / c_t^W$. By (13) and the definition of $\bar{\tau}$ this follows from $(N - 1) \bar{\tau}^2 > \sum_{j \neq i} (\tau^j)^2$.

We now show that taxes cannot be negative. Suppose to the contrary that $\exists u$ such that $\tau_u < 0$. Then, to satisfy (18), $\exists v$ such that $\tau_v > 0$ and $s_v > 0$. We demonstrate that a marginal decrease in τ_v accompanied by a marginal increase in τ_u so that (18) is satisfied increases indirect utility (17). By $\tau_v^i \in [0, \tau_{Nv}^*]$ and Proposition 1, $\partial s_v^i / \partial \tau_v^i > 0$; hence, by (17) and (18) it is sufficient to show that

$$\beta^v (\partial s_v^i / \partial \tau_v^i) (\partial u^i / \partial \tau_u^i) > \beta^u (\partial s_u^i / \partial \tau_u^i) (\partial u^i / \partial \tau_v^i). \quad (38)$$

Differentiating (17), using (13), yields

$$\partial u^i / \partial \tau_t^i = (\partial H^i / \partial \tau_t^i) / H^i - \beta^t (1 - \beta) \phi \tau_t / N c_t, \quad t \in \mathbb{Z}_+, \quad (39)$$

where $H^i \equiv \sum_{t=0}^{\infty} \beta^t (w_t - \tau_t^i) / c_t$. By (13), we have

$$\partial H^i / \partial \tau_t^i = -\beta^t [N c_t - \phi \tau_t (w_t - \tau_t^i)] / (N c_t^2), \quad t \in \mathbb{Z}_+. \quad (40)$$

By (13) and the definition of s_t^i , at a symmetric outcome $(w_t - \tau_t) / c_t = (c_t - c_t s_t) / c_t = 1 - s_t$. Thus, $H = 1 / (1 - \beta)$ and at a symmetric outcome

$$\partial u^i / \partial \tau_t^i = -\beta^t (1 - \beta) (N + \phi \tau_t s_t) / (N c_t), \quad t \in \mathbb{Z}_+. \quad (41)$$

Using (37) (at a symmetric outcome) and (41), we have that (38) is true if $(N + \phi \tau_u s_u) / \tau_u < (N + \phi \tau_v s_v) / \tau_v$. As the left-hand side of this inequality is strictly negative and the right-hand side is strictly positive, it must be true.

Proof of Proposition 3. We first find the remaining relevant derivatives for the optimisation problem. By (40), at a symmetric outcome

$$\frac{\partial^2 H^i}{\partial (\tau_t^i)^2} = \frac{\beta^t \phi N c_t (1 - s_t) - 2 \tau_t A_t}{N^2 c_t^2}, \quad \frac{\partial^2 H^i}{\partial \tau_t^i \partial \tau_s^i} = 0, \quad s \neq t, \quad s, t \in \mathbb{Z}_+, \quad (42)$$

where $A_t \equiv N + \phi\tau_t s_t - \phi\tau_t$. Hence, by (39), at a symmetric outcome

$$\frac{\partial^2 u^i}{\partial \tau_t^2} = -\frac{\beta^t (1-\beta)}{N^2 c_t^2} [D_t + \beta^t (1-\beta) A_t^2], \quad \frac{\partial^2 u^i}{\partial \tau_t^i \partial \tau_s^i} = -\frac{(1-\beta)^2 \beta^{t+s} A_t A_s}{N^2 c_t c_s}, \quad s \neq t; t \in \mathbb{Z}_+, \quad (43)$$

where $D_t \equiv \phi(Nc_t s_t + 2\tau_t A_t + \phi\tau_t^2)$. By (18) and (37), at a symmetric outcome

$$\frac{\partial B^i}{\partial \tau_t^i} = \frac{\beta^t C_t}{Nc_t}, \quad \frac{\partial^2 B^i}{\partial (\tau_t^i)^2} = \beta^t \phi \frac{Nc_t (s_t - N) + 2\tau_t C_t}{N^2 c_t^2}, \quad \frac{\partial^2 B^i}{\partial \tau_t^i \partial \tau_s^i} = 0, \quad s \neq t; t \in \mathbb{Z}_+, \quad (44)$$

where $C_t \equiv N + \phi\tau_t s_t - N\phi\tau_t$.

The Lagrangian for the government's optimisation problem is $L^i = u^i + \lambda^i B^i$, where λ^i is the multiplier. The first-order conditions $\partial u^i / \partial \tau_t^i + \lambda^i \partial B^i / \partial \tau_t^i = 0$ imply

$$\frac{\partial u^i}{\partial \tau_t^i} \frac{\partial B^i}{\partial \tau_0^i} = \frac{\partial B^i}{\partial \tau_t^i} \frac{\partial u^i}{\partial \tau_0^i}, \quad t \in \mathbb{Z}_{++}. \quad (45)$$

Substituting in the first derivatives from (41) and (44) into (45) and evaluating at a symmetric outcome yields

$$\frac{N + \phi\tau_t s_t}{\tau_t} = \frac{N + \phi\tau_0 s_0}{\tau_0}, \quad t \in \mathbb{Z}_{++}. \quad (46)$$

By (13) and the definitions of w_t , g_t and s_t^i we can write $c_t = c_t(\tau) = w_t - g_t - \phi\tau^2/2$ and $s_t = s_t(\tau) = (\tau - \phi\tau^2/2 - g_t) / c(\tau)$, $t \in \mathbb{Z}_+$.

Lemma 1. The function $\Phi_t(\tau) \equiv [N + \phi\tau s_t(\tau)] / \tau$ is strictly decreasing on $(0, \tau_{Nt}^*)$.

Proof of Lemma 1. Differentiating yields that this is true iff $v_t(\tau) \equiv \phi\tau^2[1 - \phi\tau + \phi\tau s_t(\tau)] - Nc_t(\tau) < 0$. As v_t is decreasing in N it is sufficient to show this for $N = 1$. By the definition of $\bar{\tau}$, this is the case if $\psi_t(\tau) \equiv \tau^4 - 4w_t\tau^3 + 4\bar{\tau}^2\tau^2 - \bar{\tau}^4 < 0$. We have $\psi_t(0) < 0$ and $\psi(\tau_{1t}^*) < 0$; hence, to show $\psi < 0$ on $[0, \tau_{Nt}^*] \subseteq [0, \tau_{1t}^*]$, it is sufficient to show that ψ_t has no interior maximum on $[0, \tau_{1t}^*]$. Solving $\partial\psi_t/\partial\tau = 0$ and requiring $\partial^2\psi_t/\partial\tau^2 < 0$ yields $\tau = \left(3w_t - \sqrt{9w_t^2 - 8\bar{\tau}^2}\right) / 2 > \tau_{1t}^*$; hence no interior maximum exists. ■

By Lemma 1, given τ_0 , a τ_t that solves (46) is unique; hence, $\tau_t = \tau_1$, $t \in \mathbb{Z}_{++}$.

Second-order conditions require the quadratic form $\sum_{t=0}^{\infty} \sum_{s=0}^{\infty} (\partial^2 L^i / \partial \tau_t^i \partial \tau_s^i) d\tau_t^i d\tau_s^i$ to be negative where $\sum_{t=0}^{\infty} (\partial B^i / \partial \tau_t^i) d\tau_t^i = 0$, evaluated at a symmetric solution to the first-order conditions, except at $d\tau_t^i = 0$, $t \in \mathbb{Z}_+$. Using (43), (44) and $\lambda^i = -(\partial u^i / \partial \tau_t^i) / (\partial B^i / \partial \tau_t^i)$, this is true if

$$\sum_{t=0}^{\infty} \left(\frac{\beta^t (1 - \beta) D_t}{N^2 c_t^2} + \frac{\frac{\partial u^i}{\partial \tau_t^i}}{\frac{\partial B^i}{\partial \tau_t^i}} \frac{\partial^2 B^i}{\partial (\tau_t^i)^2} \right) (d\tau_t^i)^2 + \left(\frac{1 - \beta}{N} \sum_{t=0}^{\infty} \frac{\beta^t A_t}{c_t} d\tau_t^i \right)^2 > 0. \quad (47)$$

Using $\partial B^i / \partial \tau_t^i > 0$, by Proposition 1, this is true if $\beta^t (1 - \beta) D_t \partial B^i / \partial \tau_t^i + N^2 c_t^2 (\partial u^i / \partial \tau_t^i) \partial^2 B^i / \partial (\tau_t^i)^2 \geq 0$. Using (41) and (44) we have that this is true if $N^3 c_t - N \phi \tau_t^2 - \phi^2 \tau_t^3 s_t + N \phi^2 \tau_t^3 > 0$. This is true because the left-hand side of this inequality is strictly increasing for $N > 1$ and, by $v(\tau_t) < 0$ (in the proof of Lemma 1), it is positive at $N = 1$.

Finally, we demonstrate that a unique interior equilibrium (τ_0, τ_1) exists and has $\tau_0 < \tau_1$. By (18) and (46) such an equilibrium satisfies

$$(1 - \beta) s_0(\tau_0) + \beta s_1(\tau_1) = 0 \quad (48)$$

$$\Phi_1(\tau_1) = \Phi_0(\tau_0) \quad (49)$$

By assumption (19) and Proposition 1, $\exists! \hat{\tau}$ such that $(\hat{\tau}, \hat{\tau})$ and satisfies (48). For any $\tau_0 \in (0, \hat{\tau})$, assumption (19) and Proposition 1 ensure that $\exists! \tau_1 \in (\hat{\tau}, \tau')$ such that (τ_0, τ_1) satisfies (48). The conditions of the implicit function theorem are satisfied on $(0, \hat{\tau}) \times (0, \tau')$; hence, there exists a unique continuous function $\tau^A(\tau_0)$ on $(0, \hat{\tau})$ such that $(\tau_0, \tau^A(\tau_0))$ satisfies (48). Proposition 1 ensures that $\partial \tau^A / \partial \tau_0 < 0$. The function τ^A is depicted in Figure 1 by the curve labelled *A*. Note that $\Phi_0(\tau) = \Phi_1(\tau) - \phi R_0 b_0 / c(\tau) < \Phi_1(\tau)$ if $\tau \in (0, \tau')$. By Lemma 1, if $\tau_1 \in (0, \tau')$ then $\Phi_1(\tau_1) \in (\Phi_1(\tau'), \infty)$. Also by Lemma 1, $\partial \Phi_0 / \partial \tau_0 < 0$ on $(0, \tau')$ with $\Phi_0 \rightarrow \infty$ as $\tau_0 \searrow 0$ and $\Phi_0 \rightarrow \Phi_0(\tau') < \Phi_1(\tau')$ as $\tau_0 \nearrow \tau'$. Thus, for any $\tau_1 \in (0, \tau')$, $\exists! \tau_0 \in (0, \tau_1)$ such that (τ_0, τ_1) satisfies (49). By the implicit function theorem, there exists a unique continuous function $\tau^B(\tau_1)$ on

$(0, \tau')$ such that $(\tau^B(\tau_1), \tau_1)$ satisfies (49). Clearly $\tau^B(0) = 0$ and Lemma 1 ensures that $\partial\tau^B/\partial\tau_1 > 0$. The function τ^B is depicted in Figure 1 by the curve labelled B . From Figure 1, it is clear that A and B have a unique intersection and that at the intersection $\tau_0 < \tau_1$. As s_t is strictly increasing and Φ_t is strictly decreasing on $[0, \tau_{1t}^*]$, $t = 0, 1$, (and thus any "continuation" of curve A would slope down and any "continuation" of curve B would slope up) there can be no other equilibria. This completes the proof.

Proof of Proposition 4. This follows trivially from (46).

Proof of Proposition 5. By (49), $\tau_1 > \tau_0$ and Lemma 1 of Proposition 3, an increase in N rotates curve B in Figure 1 clockwise. This yields the result.

Proof of Proposition 6. Let $\tau_t \in [0, 1)$, $t \in \mathbb{Z}_+$. Differentiating (29) and (30) yields

$$\begin{aligned} \frac{\partial u}{\partial \tau_t} &= -\frac{\beta^t \alpha (1 - \alpha) \tau_t}{(1 - \tau_t)(1 - \alpha \tau_t)} < 0, \quad t \in \mathbb{Z}_+ \\ \frac{\partial B}{\partial \tau_0} &= (1 - \beta) \alpha \left[1 - \frac{\gamma}{(1 - \tau_0)^2} \right], \quad \frac{\partial B}{\partial \tau_t} = \beta^t (1 - \beta) \alpha, \quad t \in \mathbb{Z}_{++} \\ \frac{\partial^2 u}{\partial \tau_t^2} &= -\frac{\beta^t \alpha (1 - \alpha \tau_t^2)}{(1 - \tau_t)^2 (1 - \alpha \tau_t)^2} < 0, \quad \frac{\partial^2 u}{\partial \tau_t \partial \tau_s} = 0, \quad s \neq t, \quad s, t \in \mathbb{Z}_+ \\ \frac{\partial^2 B}{\partial \tau_0^2} &= -\frac{2(1 - \beta) \alpha \gamma}{(1 - \tau_0)^3} < 0, \quad \frac{\partial^2 B}{\partial \tau_t^2} = \frac{\partial^2 B}{\partial \tau_t \partial \tau_s} = 0, \quad s \neq t, \quad t \in \mathbb{Z}_{++}, \quad s \in \mathbb{Z}_+, \end{aligned} \quad (50)$$

where $\gamma \equiv (1 - \alpha) R_0 b_0 / \alpha$.

As all of the cross-partial derivatives are zero, the second-order conditions are satisfied if $(\partial B / \partial \tau_t)(\partial^2 u / \partial \tau_t^2) - (\partial u / \partial \tau_t)(\partial^2 B / \partial \tau_t^2) < 0$, $t \in \mathbb{Z}_+$. By (50) and $\partial B / \partial \tau_t > 0$, which follows from an argument similar to that in Proposition 2, this is clearly true.

The first-order conditions are (30) and $\Psi_t(\tau_t) = \Psi_0(\tau_0)$, $t \in \mathbb{Z}_{++}$, where $\Psi_t(\tau_t) \equiv (\partial u / \partial \tau_t) / (\partial B / \partial \tau_t)$, $t \in \mathbb{Z}_+$. As Ψ_t is strictly decreasing and $\Psi_t(\tau_t) = \Psi_1(\tau_1)$, $t \in \mathbb{Z}_{++}$, it must be that $\tau_t = \tau_1$, $t \in \mathbb{Z}_{++}$ and

$$\Psi_1(\tau_1) = \Psi_0(\tau_0), \quad t \in \mathbb{Z}_{++}. \quad (51)$$

As in Proposition 3, (31) and (50) ensure that there exists a unique continuous strictly

decreasing function $\tau^A(\tau_0)$ on $(0, \hat{\tau})$ such that $(\tau_0, \tau^A(\tau_0))$ satisfies (30) (when $\tau_t = \tau_1$, $t \in \mathbb{Z}_{++}$), where $\hat{\tau}$ is the unique constant tax that satisfies (30). By (50) $\Psi_0(\tau) < \Psi_1(\tau)$ when $R_0 b_0 > 0$ if $\tau \in (0, \tau')$. By (50), if $\tau_1 \in (0, \tau')$ then $\Phi_1(\tau_1) \in (\Phi_1(\tau'), 0)$. Also by (50), $\partial\Phi_0/\partial\tau_0 < 0$ on $(0, \tau')$ with $\Phi_0 \rightarrow 0$ as $\tau_0 \searrow 0$ and $\Phi_0 \rightarrow \Phi_0(\tau') < \Phi_1(\tau')$ as $\tau_0 \nearrow \tau'$. Thus, for any $\tau_1 \in (0, \tau')$, $\exists! \tau_0 \in (0, \tau_1)$ such that (τ_0, τ_1) satisfies (51). By the implicit function theorem, there exists a unique continuous function $\tau^B(\tau_1)$ on $(0, \tau')$ such that $(\tau^B(\tau_1), \tau_1)$ satisfies (51). Clearly $\tau^B(0) = 0$ and (50) ensures that $\partial\tau/\partial\tau_1 > 0$. The geometric argument of Proposition 3, using Figure 1 holds and there exists a unique solution to the optimisation problem and it has $\tau_1 > \tau_0$.

Proof of Proposition 7. Working backward, in period 1 the government's Lagrangian is $u_1 + \lambda(s_1^c + \beta s_2)$, where $u_1 \equiv \ln c_1 + \beta \ln c_2$, consumption is given by (13), saving is as defined in (18) and $s_1^c \equiv s_1 - R_1 b_1/c_1$.

The Euler equation is given by (5) for $t = 1$. Optimality requires

$$\frac{1 + \phi\tau_1 s_1^c}{\tau_1} - \frac{1 + \phi\tau_2 s_2}{\tau_2} = 0 \quad (52)$$

$$s_1^c + \beta s_2 = 0 \quad (53)$$

$$\begin{vmatrix} L_{11} & L_{12} & \frac{\partial s_1^c}{\partial \tau_1} \\ L_{12} & L_{22} & \frac{\partial s_2}{\partial \tau_2} \\ \frac{\partial s_1^c}{\partial \tau_1} & \frac{\partial s_2}{\partial \tau_2} & 0 \end{vmatrix} > 0. \quad (54)$$

Solving (54) implies

$$\Delta \equiv (1 + \beta) \phi \frac{\partial s_1^c}{\partial \tau_1} \frac{\partial s_2}{\partial \tau_2} - \frac{\beta}{\tau_1^2} \frac{\partial s_2}{\partial \tau_2} - \frac{1}{\tau_2^2} \frac{\partial s_1^c}{\partial \tau_1} < 0. \quad (55)$$

Differentiating (52) and (53) yields

$$\frac{d\tau_1}{d(R_1 b_1)} = \frac{1}{c_1 \Delta} \left[(1 + \beta) \phi \frac{\partial s_2}{\partial \tau_2} - \frac{1}{\tau_2^2} \right], \quad \frac{d\tau_2}{d(R_1 b_1)} = -\frac{1}{c_1 \Delta \tau_1^2}. \quad (56)$$

Differentiating the indirect utility function $u_1 = u_1(R_1 b_1)$, using (56) yields

$$\frac{du_1}{d(R_1 b_1)} = -\frac{\phi\tau_1}{c_1^2} \left(\frac{\partial s_1^c}{\partial \tau_1} \right)^{-1}. \quad (57)$$

Differentiating (5) at $t = 0$ yields

$$\frac{\partial(R_1 b_1)}{\partial \tau_0} = \frac{b_1 c_1}{c_0} \frac{\phi\tau_0}{\phi\tau_1 \frac{d\tau_1}{d(R_1 b_1)} b_1 + \beta c_0}, \quad \frac{\partial(R_1 b_1)}{\partial b_1} = \frac{c_1}{\phi\tau_1 \frac{d\tau_1}{d(R_1 b_1)} b_1 + \beta c_0}. \quad (58)$$

In period 0 the policy maker maximises $\ln c_0 + \beta u_1(R_1 b_1)$ subject to (13) and (18) at $t = 0$. Optimality requires

$$\frac{\phi\tau_0}{c_0} = \beta \frac{du_1}{d(R_1 b_1)} \left[\frac{\partial(R_1 b_1)}{\partial \tau_0} - (1 - \phi\tau_0) \frac{\partial(R_1 b_1)}{\partial b_1} \right]. \quad (59)$$

Substituting in (57) and (58) into (59) yields

$$-\frac{1}{\Delta} \frac{\phi s_0}{\tau_1^2} \frac{\partial s_2}{\partial \tau_2} = \frac{1 + \phi\tau_0 s_0}{\tau_0} - \frac{1 + \phi\tau_1 s_1}{\tau_1}. \quad (60)$$

This implies that $(1 + \phi\tau_0 s_0)/\tau_0 > (1 + \phi\tau_1 s_1)/\tau_1 \Rightarrow \tau_0 < \tau_1$.

Proof of Proposition 10. Let the initial period be denoted by $t = 0$. We have $\tau_0 < 1/\phi$. If this were not true, then with constant taxes welfare could be improved by moving to the lower tax (on the other side of the Laffer curve) that produces the same surplus.

Suppose that the coordinated marginal fall in the initial tax $d\tau_0 < 0$ is financed by a sequence of future tax changes $\{d\tau_t\}_{t=1}^{\infty}$ such that $dc_t = dc$, $t \in \mathbb{Z}_{++}$. Differentiating (32) and evaluating at the initial steady state yields that a strict increase in utility requires

$$dc_0 + \beta dc / (1 - \beta) > 0. \quad (61)$$

Differentiating (35) and evaluating at a steady state yields

$$\frac{1 - \phi\tau}{\theta} \sum_{t=0}^{\infty} \beta^t d\tau_t - \left(s - \frac{R_0 b_0}{c} \right) dc_0 - \frac{\beta s dc}{1 - \beta} = 0. \quad (62)$$

At a steady state, $R_t = 1/\beta$. Thus evaluating (35) at the steady state yields $s = (1 - \beta) b_0 / (\beta c)$. Substituting this into (62) yields

$$(1 - \phi\tau) \sum_{t=0}^{\infty} \beta^t d\tau_t + \theta b_0 dc_0 / c - \theta b_0 dc / c = 0. \quad (63)$$

Differentiating (34) and evaluating at the steady state yields

$$dR_1 = \theta (dc - dc_0) / (\beta c), \quad dR_t = 0, \quad t = 2, 3, \dots \quad (64)$$

Differentiating (36), employing $\partial f_t / k_t = R_t$ and $dk_t / dR_t = -k_t / (\alpha R_t)$, substituting in (64) and evaluating at a steady state yields

$$\begin{aligned} \phi\tau d\tau_0 &= \theta (dc - dc_0) k_1 / (\alpha c) - dc_0 \\ \phi\tau d\tau_1 &= -\theta (dc - dc_0) k_1 / (\alpha \beta c) - dc \\ \phi\tau d\tau_t &= -dc, \quad t = 2, 3, \dots \end{aligned} \quad (65)$$

Substituting (65) into (63) and using $b_0 > 0$ yields that utility rises if and only if

$$\frac{1 - \phi\tau}{\phi\tau} \frac{\beta}{1 - \beta} + \frac{b_0}{c} > 0. \quad (66)$$

This follows from $\tau < 1/\phi$.

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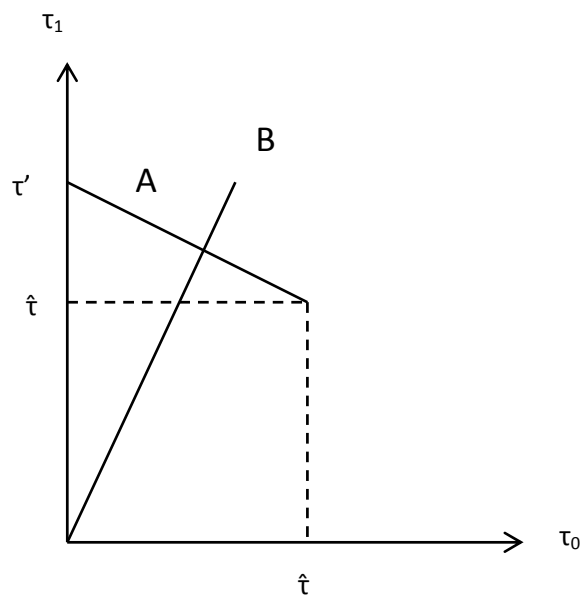


Figure 1