MSc Financial Economics
Speculative Attacks: First Generation Models

Anne Sibert

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Consider our simple discrete-time model monetary approach model with perfect foresight. The equilibrium has

$$m_t - e_t = y^* - \alpha\left(e_{t+1} - e_t\right). \tag{1}$$

Suppose that the government wants to peg the exchange rate at $\bar{e}$. Substituting this into the above equation yields

$$m_t - \bar{e} = y^* - \alpha\left(\bar{e} - \bar{e}\right) \Rightarrow m_t = y^* + \bar{e}. \tag{2}$$

The money supply becomes *endogenous* and must be fixed as well. The intuition is that with a pegged exchange rate, the expected depreciation of the domestic currency is zero. Hence, uncovered interest parity implies the nominal interest rate must equal the foreign nominal interest rate. Purchasing power parity implies the price equals the pegged exchange rate. Thus, the only variable left to adjust is the money supply.

Another possibility beside a fixed exchange rate is a crawling peg. Suppose the government sets

$$e_{t+1} - e_t = \mu. \tag{3}$$

Then, equation (1) implies

$$m_t = y^* - \alpha\mu + e_t. \tag{4}$$

Since equation (4) must hold for every value of t, it must hold for $t+1$. Thus, we also have

$$m_{t+1} = y^* - \alpha\mu + e_{t+1}. \tag{5}$$

Thus, subtracting the left-hand side of equation (4) from the left-hand side of equation (5) and the right-hand side of equation (4) from the right-hand side of equation (5), we have

$$m_{t+1} - m_t = y^* - \alpha\mu + e_{t+1} - \left(y^* - \alpha\mu + e_t\right) = e_{t+1} - e_t = \mu. \tag{6}$$

The result is that the money supply is endogenous and must also grow at rate μ .

1. The central bank balance sheet

Recall that equilibrium in the money market is given by:

$$m_t - e_t = y^* - \alpha\dot{e}_t, \tag{7}$$

where the dot denotes a time derivative.

We will suppose that the government runs a deficit and requires the central bank to monetise it. The central bank must peg the exchange rate at $\bar{e}$. The money supply is given by[1]

$$M_t = B_t^h + B_t^f \tag{8}$$

where M_t is the home money supply, B_t^h is the central bank's holdings of home bonds and B_t^f is the central bank's holdings of foreign bonds (foreign reserves). To monetise a deficit, the central bank buys the home bonds issued by the home government. Thus, home bonds (held by the central bank) go up and money in circulation goes up. But, to keep the exchange rate pegged, it must keep the money supply constant. Thus, it must intervene in the foreign exchange market. It sells its foreign bonds for home currency. This causes the central bank's holdings of foreign bonds to fall and so does money in circulation.

Suppose that monetising the deficit requires that

$$\dot{b}_t^h = \dot{B}_t^h / B_t^h = \mu > 0. \tag{9}$$

To maintain the pegged exchange rate requires

$$\dot{m}_t = \dot{M}_t = 0. \tag{10}$$

This requires

$$\dot{B}_t^f = -\dot{B}_t^h. \tag{11}$$

Clearly, this is not sustainable. Eventually $B_t^f = 0$: there are no more reserves. I will assume that the central bank devalues the currency at this point. In practice, it may devalue while it still has some reserves, or it might borrow additional reserves. I also assume that once the currency is devalued, the private sector believes that the government will let the exchange rate float for ever after.

The result is a speculative attack. The government will have a strictly positive stock of reserves right up until the instant it has to abandon its peg.

The key result we need to demonstrate this is that, if there is perfect foresight, the exchange rate cannot "jump". That is, the path of the exchange rate must be continuous.

Recall that the uncovered interest parity condition in the continuous time model is

$$i_t = i^* + \dot{e}_t. \tag{12}$$

The rate of depreciation of the home currency must equal the interest differential. This is finite; hence, the derivative of the exchange rate over time is always some finite number. As a consequence, the path of the exchange rate must be continuous.

What we want to show is that reserves vanish instantly. That is, the exchange rate does not jump when the devaluation occurs, but reserves do.

Consider what happens after the the devaluation occurs and the currency is allowed to float. We know that the exchange rate depends upon the current and all future values of the money supply. That the exchange rate was once pegged and that there was a devaluation is irrelevant. The money supply is now equal to the home bond component

[1] Recall that capital letters are levels of variables and that lower case letters are logarithms so that, for example, $m = \ln M$.

as reserves have gone to zero. This home bond component is growing at rate μ and, thus, so is the money supply. This is going to mean that the exchange rate is rising steadily over time, just as we saw in the discrete-time case. So, in mathematical terms, when the exchange rate was pegged, its derivative was obviously zero. But, from the moment of the attack, the derivative of the exchange rate is a positive number. That is, at the instant of the attack, the derivative of the exchange rate "jumps" from being zero to being a strictly positive number.

Consider the equilibrium condition again:

$$m_t - e_t = y^* - \alpha \dot{e}_t. \tag{13}$$

We have that at the instant of the collapse $\dot{e}_t$ jumps up. Thus, the left-hand size of equation (13) must jump down. As a consequence, it must also be the case that at the instant of the collapse, the right-hand side of equation (13) jumps down. Thus, we need either m_t to jump down or e_t to jumpt up. We have established, however, that the exchange rate cannot jump. So, it must be that the money supply jumps down at the instant of the attack. Up until the attack, the money supply is made up of the domestic bond component and foreign reserves. We now that the domestic bond component cannot jump: we assume that it grows at the constant rate μ. Hence, at the moment of attack, reserves jump down.The key result we use is that the exchange rate cannot jump when the peg is abandoned. Thus, the peg collapses at the instant that the pegged exchange rate equals the value that the exchange rate would take if it floated. That is, it equals the value that the exchange rate would take if reserves equalled zero and the central bank no longer intervened to support the currency.

Our first step is to find a formula for the exchange rate when reserves equal zero. That is, we want to find the exchange rate when $\dot{m}_t = \dot{b}_t^h = \mu$. The simple way to do this is to guess the form of the equation for the exchange rate. We know that the exchange rate depends on the current and all future values of the money supply. Future values of the money supply depend on the current money supply and the parameter μ. Hence, we guess that the exchange rate is a linear function of the current money supply.

$$e_t = c_0 + c_1 m_t = c_0 + c_1 b_t^h. \tag{14}$$

where c_0 and c_1 are constants to be determined. If this is true, then

$$\dot{e}_t = c_1 \dot{m}_t = c_1 \dot{b}_t^h = c_1 \mu. \tag{15}$$

Substituting equations (14) and (15) and $m_t = b_t^h$ into equation (13) yields

$$b_t^h - \left(c_0 + c_1 b_t^h\right) = y^* - \alpha c_1 \mu. \tag{16}$$

Gathering terms yields

$$\left(1 - c_1\right) b_t^h = c_0 + y^* - \alpha c_1 \mu. \tag{17}$$

For this to be true for every possible value of domestic credit, we need

$$c_1 = 1, c_0 = \alpha\mu - y^*. \tag{18}$$

Substituting equation (18) into equation (14) yields

$$e_t = \alpha\mu - y^* + b_t^h. \tag{19}$$

We call this exchange rate the *shadow floating exchange rate*. To find the instant T that the exchange rate collapses, and use equation (19) to find e_T. Then, we set $e_T = \bar{e}$ and solve for T. Evaluating (19) at $t = T$ yields

$$\bar{e} = \alpha\mu - y^* + b_T^h. \tag{20}$$

To finish this, we need to solve for b_T^h as a function of T and b_0^h. To do this, we use the differential equation (9). This differential equation is *separable* and easy to solve.

By equation (9),

$$\frac{db_t^h}{dt} = \mu \Rightarrow db_t^h = \mu dt. \tag{21}$$

Integrating from 0 to T yields

$$\begin{aligned} \int_0^T db_t^h &= \mu \int_0^T dt \Rightarrow b_t^h\big|_0^T = \mu t\big|_0^T \Rightarrow \\ b_T^h &= b_0^h + \mu T. \end{aligned} \tag{22}$$

Substituting this into equation (20) yields

$$\bar{e} = \alpha\mu - y^* + b_0^h + \mu T. \tag{23}$$

Thus,

$$T = \frac{\bar{e} + y^* - \alpha\mu - b_0^h}{\mu}. \tag{24}$$

This makes sense, the higher is $\bar{e}$ (the lower is the value of the currency under the peg) the longer the peg will last. The higher is μ (the bigger is the fiscal deficit), the shorter the peg will last.

Note that nothing ensures that T is strictly positive. The way to think of this is that at date 0, the government announces that it is pegging the exchange rate and that domestic credit will grow at μ. The public believes that domestic credit will grow at μ and that the government will defend the exchange rate until it runs out of reserves. If $\bar{e}$ is sufficiently small or the growth in domestic credit is sufficiently large, then the exchange rate collapses immediately. A possible example of thie is Iceland. On 7 Oct 2008, Iceland attempted to peg the króna at 131 kr. per euro. It abandoned the peg the next day and the króna depreciated to 340 kronur/euro.

This timing of a speculative attack is illustrated in Figure 1.

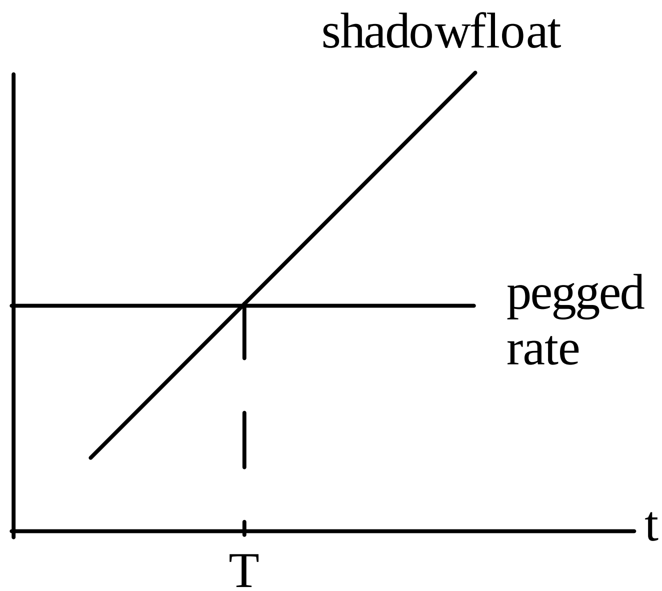
What does the path of reserves look like? Before time T, equation (23) implies

$$m_t = \bar{e} + y^*. \tag{25}$$

Thus, by equation (8)

$$\begin{aligned} \ln\left(B_t^h + B_t^f\right) &= \bar{e} + y^* \Rightarrow \\ B_t^h + B_t^f &= \exp\left(\bar{e} + y^*\right). \end{aligned} \tag{26}$$

By equation (22),



$$\begin{aligned} b_T^h &= b_0^h + \mu T \Rightarrow \ln B_T^h = \ln B_0^h + \mu T \Rightarrow \\ \ln\left(\frac{B_T^h}{B_0^h}\right) &= \mu T \Rightarrow B_T^h = \exp\left(\mu T\right) B_0^h. \end{aligned} \tag{27}$$

Substituting this into equation (27) into (26) implies[2]

$$\lim_{t \to T} B_t^f = \exp\left(\bar{e} + y^*\right) - \exp\left(\mu T\right) B_0^h. \tag{28}$$

Substituting equation (24) into equation (28) yields

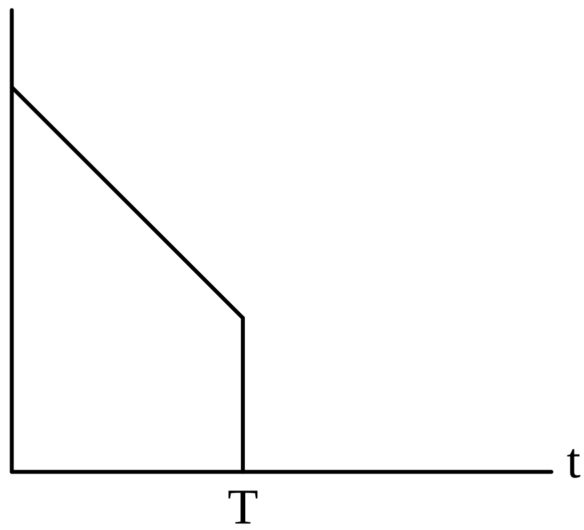
$$\begin{aligned} \lim_{t \to T} B_t^f &= \exp\left(\bar{e} + y^*\right) - \exp\left(\bar{e} + y^* - \alpha\mu - b_0^h\right) B_0^h \\ &= \exp\left(\bar{e} + y^*\right) - \frac{\exp\left(\bar{e} + y^* - \alpha\mu\right) B_0^h}{\exp\left(b_0^h\right)} \\ &= \exp\left(\bar{e} + y^*\right) - \exp\left(\bar{e} + y^* - \alpha\mu\right) \\ &= \exp\left(\bar{e} + y^*\right)\left[1 - \exp\left(-\alpha\mu\right)\right]. \end{aligned} \tag{29}$$

This is strictly positive if and only if

$$1 - \frac{1}{\exp\left(\alpha\mu\right)} > 0 \Leftrightarrow \exp\left(\alpha\mu\right) > 1 \Leftrightarrow \alpha\mu > 0. \tag{30}$$

This is true. Thus, right up until the instant of devaluation, there is a strictly positive stock of reserves. The path of reserves is sketched in Figure 2.

[2] We use the limit because we want to find the value of reserves the instant before T.



The speculative attack causes reserves to jump at time T.

We have analysed the "no bubble" solution here. But, it is possible to have bubbles. The formula for a bubble solution is

$$e_t = \alpha\mu - y^* + b_t^h + c \exp(t/a), \tag{31}$$

where c is any constant. [3] Proceeding as before, we have be

$$\begin{aligned} e_T &= \bar{e} = \alpha\mu - y^* + b_0^h + \mu T + c \exp(T/a) \Rightarrow && (33) \\ T + (c/\mu)\exp(T/a) &= \frac{\bar{e} - \alpha\mu + y^* - b_0^h}{\mu} && (34) \end{aligned}$$

If $c > 0$, then the attack occurs faster than it would without a bubble. If c is large enough, it can cause the attack to occur immediately.

Suppose $\mu = 0$. This is where the fundamentals do not warrant an attack on the exchange rate. Rewrite equation (33)

$$\bar{e} = -y^* + b_0^h + c \exp(T/a) \tag{35}$$

Solving yields

[3] We can verify that this satisfies the equilibrium condition (13) when reserves are zero. Differentiating yields

$$\dot{e}_t = \mu + (c/a)\exp(t/a), \tag{32}$$

Subtitute this and equation (31) into equation (13):

$$\begin{aligned} b_t^h - \left[\alpha\mu - y^* + b_t^h + c\exp(t/a)\right] &= y^* - \alpha\left[\mu + (c/a)\exp(t/a)\right] \Leftrightarrow \\ -\alpha\mu + y^* - c\exp(t/a) &= y^* - \alpha\mu - c\exp(t/a), \end{aligned}$$

which is true.

$$T = a \ln\left(\frac{\bar{e} + y^* - b_0^h}{c}\right) \tag{36}$$

A bubble can cause a speculative attack even when it is not warranted by the fundamentals. If c is large enough, the attack can be immediate.

In theory, speculative attacks can occur solely as a result of self-fulfilling expectations.